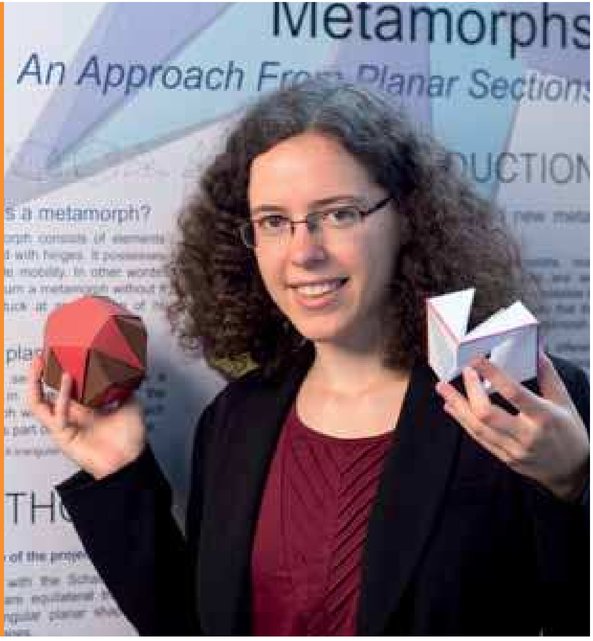

Viviane Kehl, 1994
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Rämibühl-MNG
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Würdigung durch den Experten
Dr. Bernhard Ruh

Durch einen gänzlich neuen Ansatz findet die Autorin eine Fülle von bisher unbekannten, beweglichen Körpern (sogenannten Metamorphs). Diese Körper werden mit Hilfe raffinierter geometrischer Überlegungen (z. B. mit dem Satz von Menelaos) beschrieben und klassifiziert. Neben der mathematischen Theorie werden auch interessante praktische Aspekte erörtert. Die in Englisch verfasste Arbeit zeugt insgesamt von einer hohen räumlichen Vorstellungskraft und von einem aussergewöhnlichen mathematischen, insbesondere geometrischen, Verständnis.

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Metamorphs – An Approach From Planar Sections

Introduction
A metamorph is a model that possesses full-cycle mobility, which means it performs a sequence of steps before returning in its original starting position. A planar section is a section in the cycle of the metamorph in which all elements have a face that lies in a common plane.
There are different methods to find new metamorphs: A standard method is to start with a solid and ask whether it is possible to cut parts from it in a way that the remaining parts form a metamorph.
Contrary to that, I started by looking at a shape. I wondered if it is possible to find a metamorph for which this shape forms one of its planar sections. This idea came to my mind by looking at the Schatz Cube, one of the best known metamorphs. This metamorph possesses two equilateral triangles as planar sections. I tried to generalize this special form of planar section and therefore looked at various other shapes.
So, the goal of my project was to find shapes that yield metamorphs. Specifically, I looked mostly at general triangles, but also at polygons and regular stars.
Further, I also aimed to draw sample nets and to construct metamorphs.

Methods
A book by Robert Byrnes gave me a first insight in this topic. I got in touch with him and received a copy of his proof of a theorem found by Raoul Bricard.
I started with the Schatz Cube and then looked at other triangular shapes. To have an appropriate vocabulary, I defined amazing points. (Amazing points are points in the first planar section of a triangle that yield metamorphs).
Then, starting with the dodecahedron metamorph, I also studied general regular star metamorphs.
To build a total of six metamorphs, coloured paper, glue, a cutter and geometrical equipment was needed. To connect the single tetrahedrons of the metamorphs, thick brown tape was used.
The figures were drawn by hand, scanned and then ameliorated by using GIMP. The program that colours amazing points was written in C++ with the library OpenGL.

Results
I was able to derive the conditions which amazing points fulfil. In addition, I wrote a program that shows them.
Then, I could describe some special groups of shapes that always yield a metamorph: Acute angled triangles, obtuse angled isosceles triangles, and a cyclic polygon where the midpoint of the circumscribed circle lies inside the polygon. Furthermore, I showed that there are triangles as well as polygons which do not yield metamorphs.
I derived a surprisingly lovely formula that allows the construction of nets of star metamorphs.
In the practical part, I constructed sample nets of metamorphs and built models of metamorphs myself. Each of the six models illustrates a certain part of my project. With a model next to you, it is easier to understand the theory of metamorphs. Additionally, I constructed the remaining parts of the Schatz Cube and the dodecahedron metamorph.

Discussion
The goal of finding metamorphs with certain shapes as planar sections was reached. For triangular planar sections, I even found all of the amazing points, and thus all the points that can be used to construct a metamorph. It would be nice to do this for polygonal shapes as well, but this is more difficult because there are more degrees of freedom left.
The practical goals were achieved as well. The built models turn smoothly and are mostly precise. The star metamorph is a bit less precise than the others since it does not possess right angles. Therefore, it was more difficult to construct its nets accurately.

Conclusions
I characterised metamorphs with a newly developed approach; instead of looking at solids, I looked at planar sections. This approach has been very fruitful in finding new metamorphs and describing some of their interesting properties.
I also built models that visualize the theory behind metamorphs.
Of course, I could not cover all aspects of metamorphs. They still leave a lot of interesting questions. I hope that my project will lead to further research on these fascinating objects.